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Few-nuclon physics based on chiral dynamics

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Abstract. After a brief review of results in few-nucleon physics based on conventional nuclear forces the steps towards nuclear forces in the effective field theory approach constrained by chiral symmetry are indicated. Several low-energy constants are adjusted to nucleon-nucleon phase shift values and in the three-nucleon system to the triton binding energy and the doublet *nd* scattering length. Applications based on NNLO nuclear forces to the three- and four-nucleon systems are very promissing. Finally, a new regularization scheme is described which leads to a nice convergence of the chiral expansion.

PACS. 21.30.Cb Nuclear forces in vacuum – 21.45.+v Few-body systems

1 Introduction

In recent years so called high precision NN forces have been developed [1]. They are constructed with the aim to describe the rich set of NN data up to the pion threshold very well. These forces typically depend on about 45 fit parameters and are not founded on a systematic theoretical approach. The accurate description is considered to be a prerequisite for the use of those forces in few-nucleon sytems with $A > 2$. Over the years accurate methods have also been developed to solve the Schroedinger equation for few-nucleon bound states [2]. The result is that NN forces alone underbind light nuclei as documented in Table 1. In view of the composite nature of nucleons and a relatively low lying excited state of the nucleon, the delta, a three-nucleon force caused by an intermediate delta excitation appears as a natural candidate to fill that gap of underbinding.

Thus three-nucleon force models have been developed around that picture of an intermediate delta excitation generated by two-pion exchanges [6]. Adding them to the NN forces and adjusting their parameters one can easily achieve the correct binding energy of ^{3}H (^{3}He), the first nuclei, where 3N forces are acting. Thus an enriched Hamiltonian consisting of NN and 3N forces can now be applied to predict 3N scattering observables, the binding energy of light nuclei etc. Table 2 displays the prediction [7] for the α particle binding energy using different NN and 3N force combinations. This looks promising.

Also promising are predictions for some 3N scattering observables shown in Figs. 1,2. On the other hand for certain other spin observables the addition of these 3N forces does not improve the description, as shown in Fig. 3. This reveals that the spin and momentum dependencies of 3N forces are yet by far not understood. Certainly the variety of meson exchange processes contributing to the NN forces should also show up in 3N forces, which points to a rich set of 3N forces to be explored. Recently steps in that direction have been undertaken [9] by considering three-pion

Table 1. Theoretical ground state binding energies for various light nuclei and various NN forces in comparison to experiment. The results for $A = 6$ and 8 are from [5]

	Nijm	CD Bonn	AV18	Exp
^{3}H	−7.74	−8.01	−7.6	−8.48
^{4}He	−24.98	−26.26	−24.1	−28.30
^{6}He			−23.9	−29.3
^{6}Li			−26.9	−32.0
^{8}Li			−31.8	−41.3
^{8}Be			−45.6	−56.5

Table 2. Theoretical predictions for the α-particle binding energy for various NN and 3N force contributions

2N+3N forces	^{3}H	^{4}He
CD Bonn + TM [6]	−8.48	−28.4
AV18 + TM [6]	−8.45	−28.36
AV18 + URB [6]	−8.48	−28.50
Exp	−8.48	−28.30

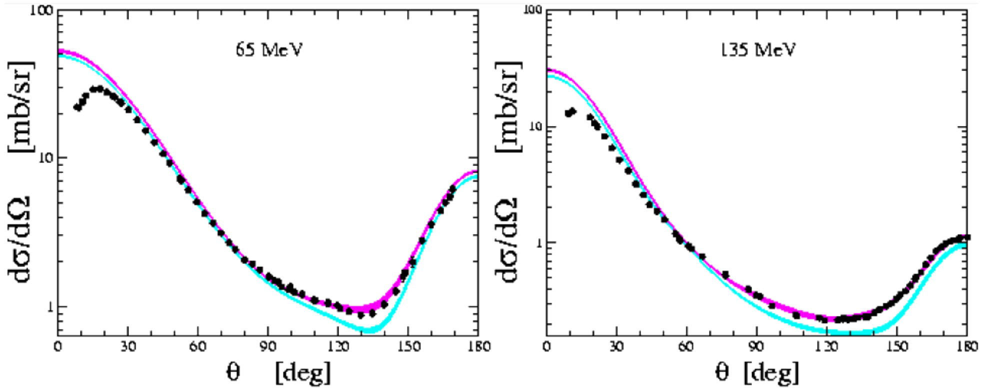

Fig. 1. Differential cross section for elastic nd scattering at $E_{\text{lab}} = 65$ und 135 MeV. The light shaded band refers to NN force predictions only, the dark shaded band includes 3N forces. Data from [3], [4]

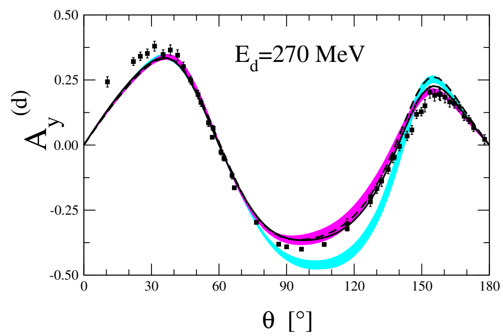

Fig. 2. Deuteron vector analysing power A_y at $E_{\text{lab}} = 135$ MeV. The solid (*dashed*) lines refer to AV18 + URBANA IX (CD Bonn + TM') nuclear forces. Data from [8]. For remaining notation see Fig. 1

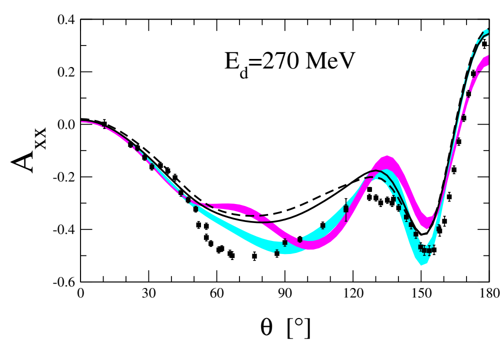

Fig. 3. Tensor analysing power A_y at $E_{\text{lab}} = 135$ MeV. For notation see Fig. 2

exchanges with intermediate deltas, which improve significantly the description of low lying states up to $A = 10$, see [10]. Nevertheless what has been and is still missing in all that work is a systematic path towards nuclear forces. This appears to change now with the effective field theory approach constrained by chiral symmetry.

2 Effective field theory approach

It is well known that the QCD Lagrangian for massless up and down quarks is invariant under global $SU(2)_L \times SU(2)_R$ transformations in the flavour space. Further there is ample evidence that this symmetry is spontaneously broken down to the isospin group $SU(2)_V$ and as consequence three massless Goldstone bosons exist, which can be identified with massless pions. The actual nonvanishing masses of the up and down quarks introduce an explicit symmetry breaking which leads to the nonvanishing pion masses. That spontaneously and explicitely broken symmetry governs also the interactions between pion and nucleon fields in an effective field theory set up. There it has to appear as a nonlinear realisation of the chiral group as worked out in [11]. In that effective field theory framework constrained by chiral symmetry the ordering scheme for the most general Lagrangian appears naturally in a low momentum regime, where the momenta stay below a certain mass scale. A convenient formulation to control low momenta for nucleons is the heavy baryon formalism, see e.g. [12]. One arrives at an effective Lagrangian for pion and nucleon fields suitable for low energy nuclear physics, which is arranged according to increasing values of the chiral dimension

$$\Delta = d + \frac{1}{2}n - 2\,. \tag{1}$$

Here d is the number of derivatives or M_π-insertions and n the number of nucleon fields for each coupling term. Chiral symmetry enforces that $\Delta \geq 0$. Besides coupling terms among pions and nucleons there occur also coupling terms among pions themselves and nucleon themselves.

To arrive at nuclear forces old-fashioned time-ordered perturbation theory has originally been applied [13,14], which, however, leads to energy-dependent forces. They are not suitable for applications to nuclear systems with > 2 nucleons. We apply a method of unitary transformations [15], where the field theoretical Hamiltonian acting in the Fock space of pions and nucleons is block-diagonalised such that the space of pure nucleon states is decoupled from the rest space (which includes pions). This decoupling and thus the derivation of the effective nucleonic Hamiltonian can be organised according to the ratio of generic external nucleonic momenta and the previously mentioned mass scale [16]. The mass scale is of the order of the ρ-mass where new physics beyond pion

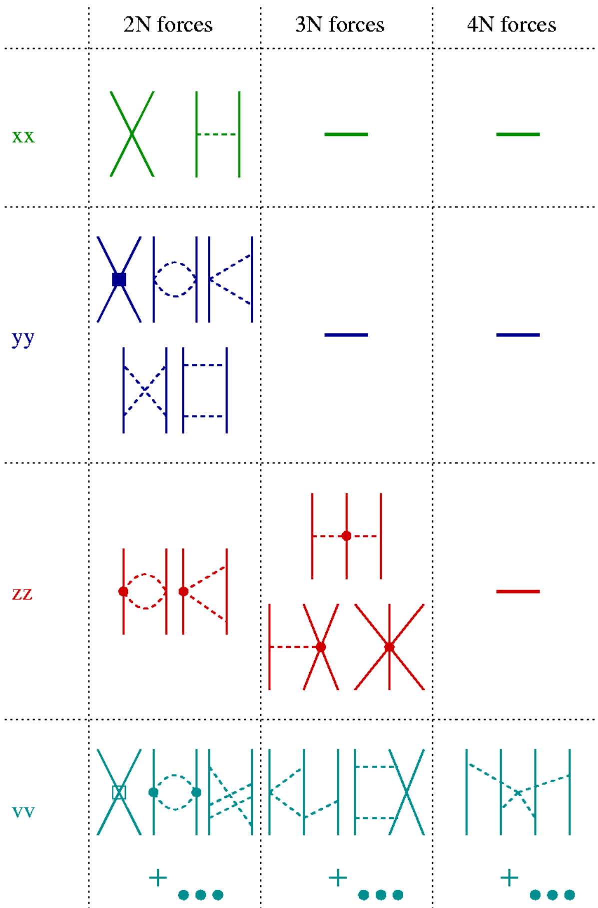

Fig. 4. Hierarchy of the nuclear forces

exchanges occurs. The resulting nuclear forces are of two types: multi-pion exchanges and a sequence of nucleonic contact forces. To each nuclear force diagram a mass dimension can be associated which is given as

$$\nu = -4 + 2N + 2L + \sum_i V_i \Delta_i \,, \qquad (2)$$

where N is the number of nucleons entering (or leaving) the diagram, L is the number of loops, V_i the number of vertices of type i and Δ_i the chiral dimension mentioned before. Because of the property $\Delta \geq 0$, ν can not be negative and starts for $N \geq 2$ at $\nu = 0$. Thus the nuclear forces can be classified according to increasing values of ν. This is schematically displayed in Fig. 4.

We see for $\nu = 0$ the one-pion exchange acompanied by NN contact forces. The contributions with $\nu = 0$ are usually called the leading order (LO) contributions. $\nu = 1$-terms do not appear due to parity conservation. Thus the next-to-leading order (NLO) terms go with $\nu = 2$ and comprise various types of two-pion exchanges and several contact forces quadratic in the momenta and M_π. At $\nu = 3$, the next-to-next-to-leading order (NNLO), there occur additional two-pion exchanges with higher order vertices, but no new contact forces. Note that all multi-pion exchanges are parameter free in the sense that the constants at the vertices, called low-energy constants (LEC's), can be taken from the analysis of the πN system. The constants going with the NN contact forces, however, have to be adjusted in the NN system. At NNLO there occur for the first time nonvanishing three-nucleon forces of three topologies: a two-pion exchange, a one-pion exchange between a NN contact interaction and the third nucleon and a pure three-nucleon contact force. The one strength parameter of the latter one has to be adjusted in the 3N system, whereas the one strength parameter at the NNNNπ vertex could in principle be determined from pion production in the NN system. Since this has not yet been worked out sufficiently well we will adjust this constant also in the 3N system (see below). Then, at NNNLO, additional two- and three-pion exchange processes and more NN contact forces contribute in the NN system. Very interestingly, at this order, a whole host of new three-nucleon forces arises, which do not include new 3N contact interactions and therefore can be expected to provide a very interesting predictive power for systems with $A \geq 3$. Finally, in that order, for the first time four-nucleon forces occur. The overview displayed in Fig. 4 nicely shows the ordering scheme that NN forces are stronger than 3N forces and 3N forces are stronger than 4N forces etc. Of course all that is restricted to a low momentum regime where the generic nucleon momenta Q are smaller than the chiral symmetry breaking scale.

The loop diagrams have to be regularized. In [18,19] we used an infinite cut-off, which is in that case equivalent to dimensional regularization, and combined the diverging terms with the contact forces. As a result at NNLO rather strong intermediate and short range attractive forces arise, which lead to a strong cut-off dependence in the D-waves and to deeply bound spurious NN states in low partial waves. Though these states lie outside the range of validity of the low-momentum theory and do not harm the low energy NN observables, they cause technical difficulties in $A > 2$ systems. For instance in the Faddeev–Yakubovsky scheme NN forces are summed up into NN t-matrix, which has poles at NN bound state energies. These spurious NN bound states appearing as additional poles have to be handled with great care which is technically not trivial [18]. In addition, they also lower the 3N binding energy because of orthogonality arguments. As a consequence the 3N forces have to provide more attraction than one is used to have in the conventional framework of nuclear forces [20]. In our first approach [18] we avoided that dynamical scenario by lowering artificially the values of the LEC's c_3 and c_4 at the N$\pi\pi$ vertices below the strengths found in the πN system. This leads to NN forces which in [18] and in the following will be denoted by NNLO* forces and which no longer support spurious NN bound states. We refer to [18] for a discussion of that procedure. Now we introduced a different regularisation scheme [21], which will be described below and restore the consistency of the LEC's c_3 and c_4 with the πN system.

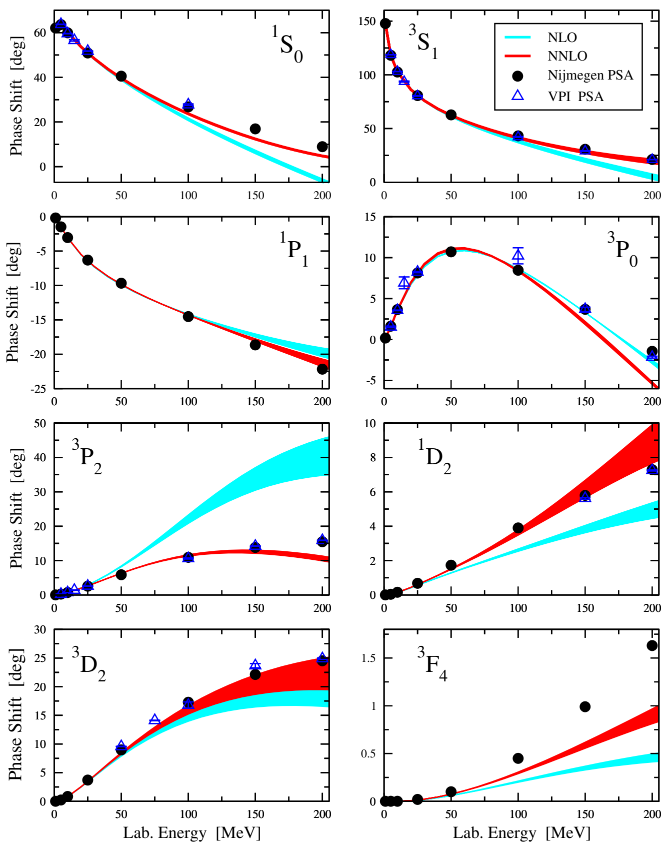

Fig. 5. NN phase shifts at NLO and NNLO

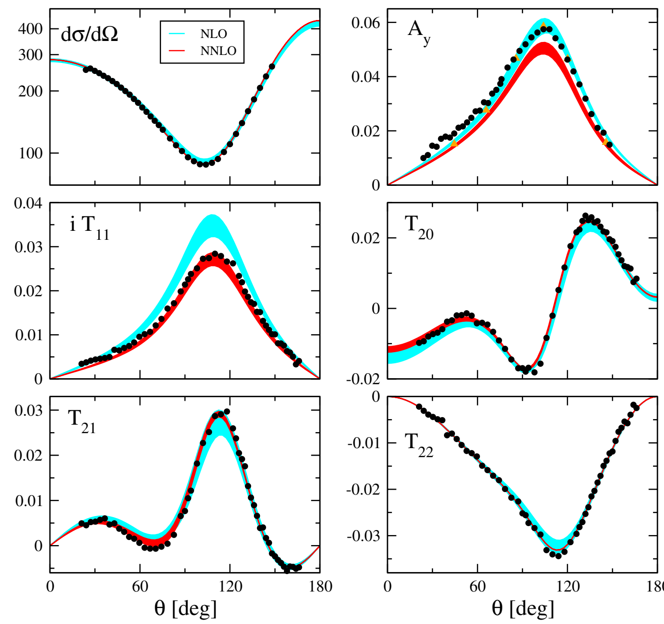

Fig. 6. *nd* elastic scattering observables at $E_{\text{lab}} = 3$ MeV

As a last step one has to adjust the forces to the low momentum regime where the theory is valid. We enforce that by introducing smooth cut off functions in the form

$$V_{\Lambda}^{\text{reg}}(\mathbf{p}',\ \mathbf{p}) = e^{-p'^4/\Lambda^4}\, V(\mathbf{p}',\ \mathbf{p}) e^{-p^4/\Lambda^4} \tag{3}$$

It turns out that

$$500 \text{ MeV } \leq \Lambda \leq 600 \text{ MeV} \tag{4}$$

is a good choice. The cut-off should be not too small in order not to cut off the pion exchange physics, and not too large in order not to enter into the domain of short range physics, which is not controlled in EFT. One expects [22] that the dependence will get weaker with increasing order in ν.

As a first step in the application the LEC's for the NN and 3N forces have to be adjusted. At LO and NLO there are 2 +7 LEC's going with the NN contact forces. They are adjusted [23] to the S- and P-wave NN phase shifts and to ϵ_1 which are known from NN phase shift analysis. The two LEC's going with the 3N forces are adjusted to the ^{3}H binding energy and the doublet nd scattering length $^2a_{nd}$ [19]. Then up to NNLO all parameters are fixed and the Hamiltonian including NN and 3N forces can be applied to predict other 3N and $A \geq 4$ observables.

In Fig. 5 we display some NN phase shifts at NLO and NNLO. We see a nice improvement in going to NNLO, which comes about by additional two-pion exchanges. In that order no new contact forces arise. Deuteron properties are also well reproduced [18].

Now we move on to predictions for 3N and 4N observables. In Figs. 6,7 we show the differential cross section in elastic nd scattering together with vector and tensor analysing powers at NLO and NNLO (this still refers to the choice NNLO*, mentioned above and will be changed in the near future). We see for both energies a nice improvement at NNLO in relation to NLO. The so called low energy A_y-puzzle [24], however, remains still open at this order and we expect that only higher order 3N forces and possibly relativistic corrections will solve that long standing problem. Altogether these results are very promising. This is also the case regarding the α-particle binding energy shown in Table 3. While at NLO there is still a rather large Λ-dependence it shrinks at NNLO and one ends up rather close at the experimental value (Please note that at this level we restricted ourselves to np forces only and had to correct for that, see [19]). Of course that shrinkage is also connected to the fact that at NNLO the ^{3}H binding energy has been adjusted. The strong correlation between ^{3}H and ^{4}He binding energies, known from investigations with conventional forces [25,7], enforces that the ^{4}He binding energy can not be far off the experimental value. There are also interesting and promising results for the $p+d \rightarrow N+N+N$ break up process and we refer the reader to [19].

Table 3. Chiral predictions for the triton- and α-particle binding energies at NLO and NNLO* compared to the experimental values, which are corrected for *np* forces

	NLO	NNLO	“Exp”
^{3}H	$-7.53\ldots-8.54$	-8.68	-8.68
^{4}He	$-23.87\ldots-29.57$	$-29.51\ldots-29.98$	-29.6

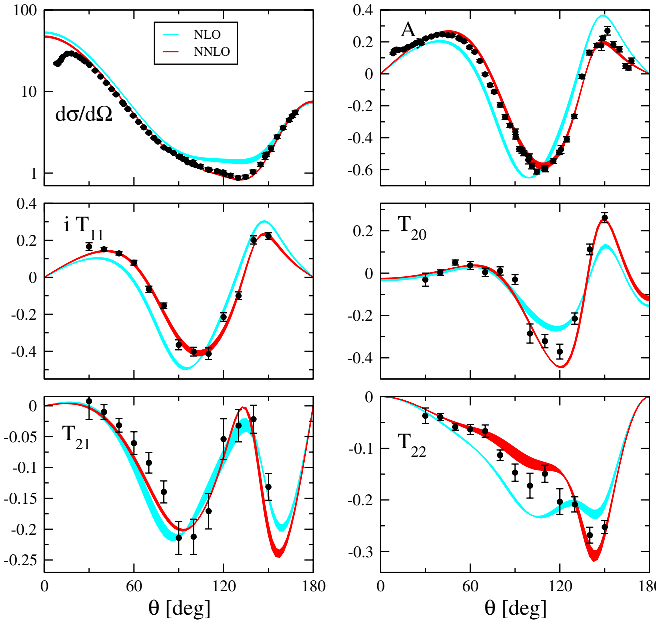

Fig. 7. nd elastic scattering observables at $E_{\rm lab} = 65$ MeV

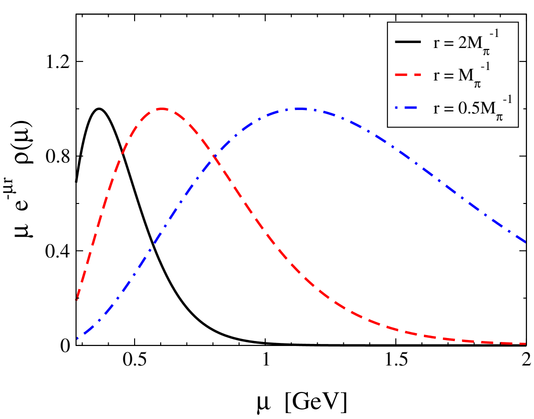

Fig. 8. The integrand of (6) for various r-values

The last step forward to be reported in this overview are recent investigations [21] on the spectral function regularisation (SFR). The two-pion exchange forces can be written as

$$V(q) = \frac{2}{\pi} \int_{2M_\pi}^{\infty} d\mu\, \mu \,\frac{\rho(\mu)}{\mu^2 + q^2}\,, \tag{5}$$

modulo subtractions. This is obviously a superposition of Yukawa interactions, where the spectral function $\rho(\mu)$ is known analytically and simply related [17] to the analytical expressions of the potential. In configuration space it results

$$V(r) = \frac{1}{2\pi^2 r} \int_{2M_\pi}^{\infty} d\mu\, \mu\, e^{-\mu r}\, \rho(\mu) \tag{6}$$

It is instructive to regard the integrand in (6) as a function of μ, the mass exchanged between the two nucleons, and this for different pair distances r. This is shown in Fig. 8 for the isoscalar central part of the subleading (i.e. NNLO) two-pion exchange potential. We see at small r-

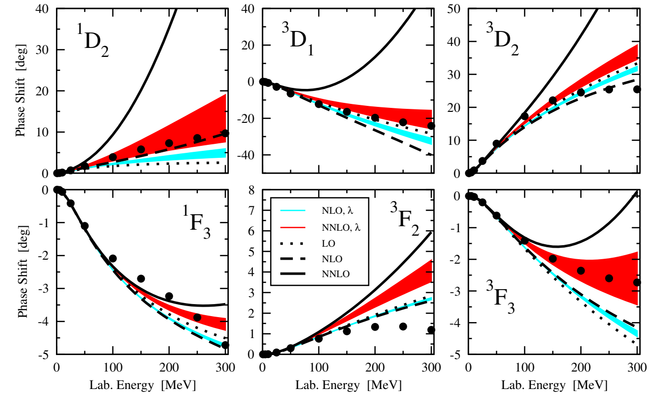

Fig. 9. Peripheral NN phase shifts at LO (*dotted line*), NLO and NNLO in dimensional regularization (*dashed and solid lines*) compared to predictions for SFR (*light and dark shaded bands*)

values large μ-components contribute in contrast to large r's where only small μ-components contribute substantially. Now the exchange of masses μ larger than say Λ belong to short range physics parametrised in this effective field theory approach by contact forces. Thus it is mandatory to cut-off the integral over μ. We do it by a sharp cut off, which introduces another parameter λ, which can be chosen in a similar range as Λ. This "long-distance" regularisation cures now the insufficient results achieved in dimensional regularisation (or infinite cut-off regularisation) at NNLO and at higher orders. We illustrate that in Fig. 9 in the case of peripheral NN phase shifts. We see the results where the loops are evaluated in dimensional regularisation, deviating drastically from the NN phase shift values, whereas using that new regularisation scheme one achieves a nice convergence in going from NLO to NNLO.

This regularization scheme is presently applied also to NNNLO where additional short range forces allow for a rather good description of the NN phase shift values up to about 200–250 MeV nucleon laboratory energy. In that order also a rich group of 3N forces occur, which is under investigation.

Summarising, this new effective field theory approach constrained by chiral symmetry is a systematic way to generate nuclear forces where NN and 3N forces are consistent. Because of the low-momentum cut-off it also allows to incorporate in a well converging manner relativistic corrections.

Moreover, since that approach is based on a Lagrangian the coupling of the photon to the pion-nucleon system is well defined and nucleonic electromagnetic current operators can be constructed which are consistent to nuclear forces, a requirement which is not sufficiently well taken care of in the conventional approach up to now. First steps in that direction have already been done [26, 27, 28].

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